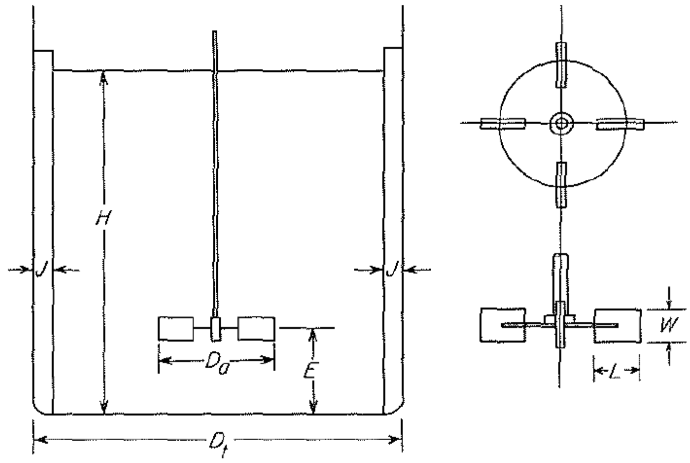
Agitation And Mixing of Liquids

Standard design of turbine impeller:

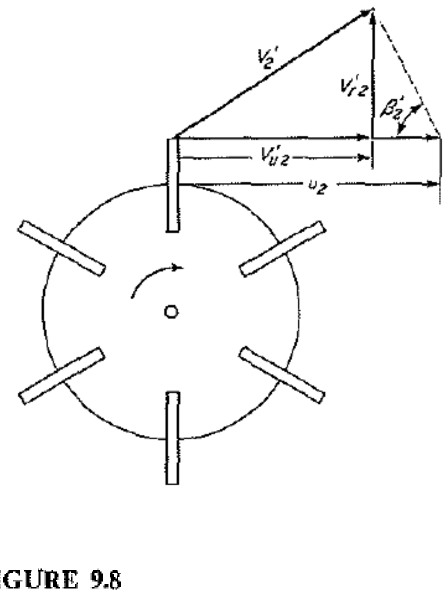


$$\frac{D_a}{D_t} = \frac{1}{3} \qquad \frac{H}{D_t} = 1 \qquad \frac{J}{D_t} = \frac{1}{12}$$

$$\frac{E}{D_t} = \frac{1}{3} \qquad \frac{W}{D_a} = \frac{1}{5} \qquad \frac{L}{D_a} = \frac{1}{4}$$

FIGURE 9.7
Measurements of turbine. (*After Rushton et al.*[42])

Flow number (N_Q):



$$V'_{u2} = ku_2 = k\pi D_a n$$

$$q = V'_{r2} A_p$$

$$A_p = \pi D_a W$$

$$V'_{r2} = (u_2 - V'_{u2}) \tan \beta'_2$$

$$q = K\pi^2 D_a^2 n W(1 - k) \tan \beta'_2$$

$$q \propto nD_a^3$$

$$N_Q \equiv \frac{q}{nD_a^3}$$

The total flow for flat-blade turbine:

$$q_T = 0.92 n D_a^3 \left(\frac{D_t}{D_a}\right)$$

FIGURE 9.8
Velocity vectors at tip of turbine impeller blade.

The flow number N_Q may be considered constant. For the design of baffled agitated vessels the following values are recommended:

For marine propellers	(square pitch)	$N_Q = 0.5$
For a four-blade 45° turbine	$(W/D_a = \frac{1}{6})$	$N_Q = 0.87$

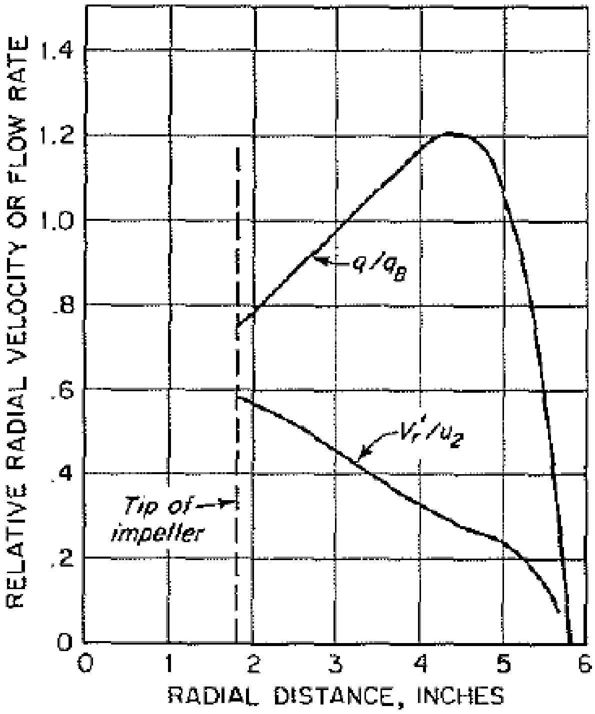


FIGURE 9.10
Radial velocity V_r'/u_2 and volumetric flow rate q/q_B in a turbine-agitated vessel. (*After Cutter.*[13])

CALCULATION OF POWER CONSUMPTION.

$$\frac{Pg_c}{n^3D_a^5\rho} = \psi\left(\frac{nD_a^2\rho}{\mu}, \frac{n^2D_a}{g}, S_1, S_2, \ldots, S_n\right)$$

$$N_P = \psi(N_{Re}, N_{Fr}, S_1, S_2, \ldots, S_n)$$

$$S_1 = \frac{D_a}{D_t} \qquad S_2 = \frac{E}{D_t} \qquad S_3 = \frac{L}{D_a}$$

$$S_4 = \frac{W}{D_a} \qquad S_5 = \frac{J}{D_t} \qquad S_6 = \frac{H}{D_t}$$

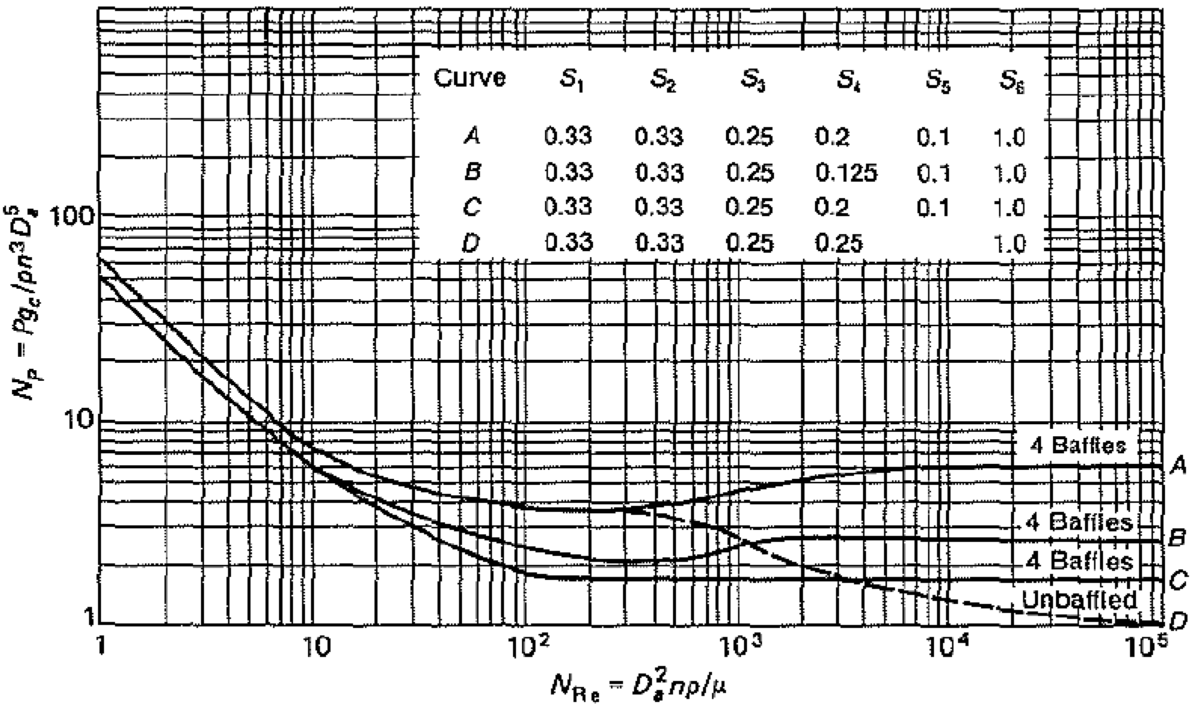


FIGURE 9.12
Power number N_p versus N_{Re} for six-blade turbines. (*After Chudacek*[11]; *Oldshue*.[35]) With the dashed portion of curve *D*, the value of N_p read from the figure must be multiplied by N_{Fr}^m.

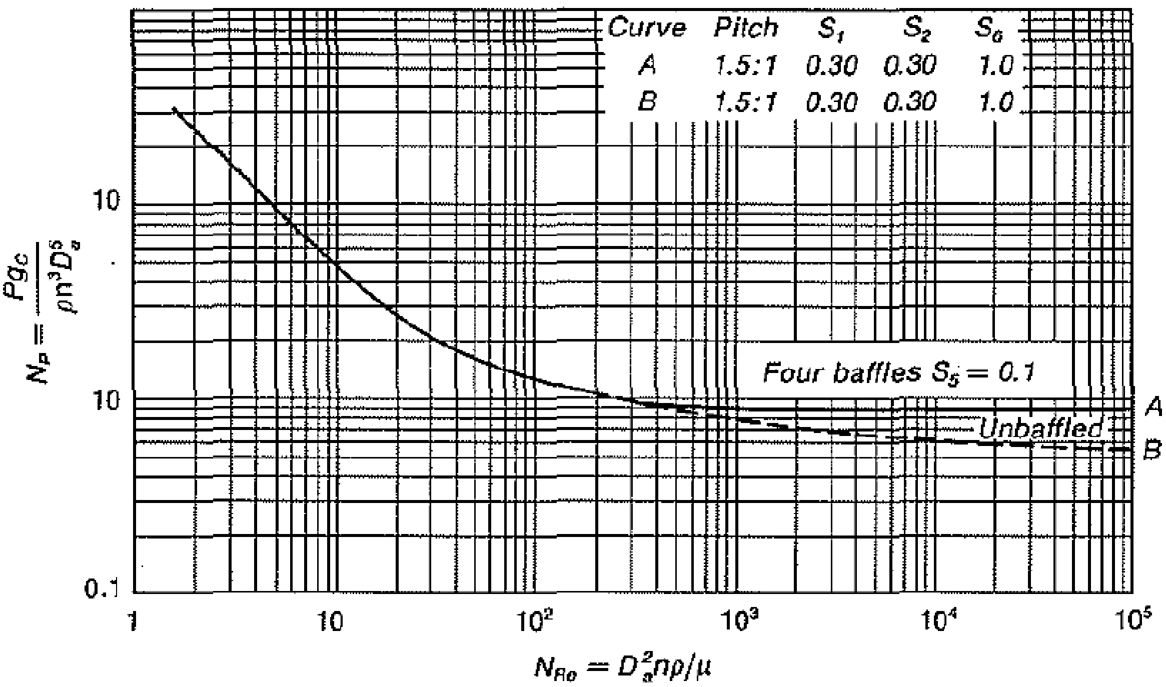


FIGURE 9.13
Power number N_p versus N_{Re} for three-blade propellers. (*After Oldshue*.[35]) With the dashed portion of curve *B*, the value of N_p read from the figure must be multiplied by N_{Fr}^m.

Unbaffled tanks:

$$\frac{N_P}{N_{Fr}^m} = \psi(N_{Re}, S_1, S_2, \ldots, S_n)$$

$$m = \frac{a - \log_{10} N_{Re}}{b}$$

TABLE 9.1
Constants a and b of Eq. (9.19)

Figure	Line	a	b
9.12	D	1.0	40.0
9.13	B	1.7	18.0

TABLE 9.2
Effect of blade width and clearance on power consumption of six-blade 45° turbines[11,39]

W/D_a, (S_4)	Clearance, S_2	K_T
0.3	0.33	2.0
0.2	0.33	1.63
0.2	0.25	1.74
0.2	0.17	1.91

TABLE 9.3
Values of constants K_L and K_T in Eqs. (9.21) and (9.23) for baffled tanks having four baffles at tank wall, with width equal to 10 percent of the tank diameter

Type of impeller	K_L	K_T
Propeller, three blades		
Pitch 1.0[40]	41	0.32
Pitch 1.5[35]	55	0.87
Turbine		
Six-blade disk[35] ($S_3 = 0.25$, $S_4 = 0.2$)	65	5.75
Six curved blades[40] ($S_4 = 0.2$)	70	4.80
Six pitched blades[39] (45°, $S_4 = 0.2$)	—	1.63
Four pitched blades[35] (45°, $S_4 = 0.2$)	44.5	1.27
Flat paddle, two blades[40] ($S_4 = 0.2$)	36.5	1.70
Anchor[35]	300	0.35

For laminar flow(N_{Re}<10):

$$N_P = \frac{K_L}{N_{Re}}$$

For fully turbulent flow (N_{Re}>10000):

$$N_P = K_T$$

POWER CONSUMPTION IN NON-NEWTONIAN LIQUIDS.

$$N_{Re,n} = \frac{nD_a^2\rho}{\mu_a} \qquad \mu_a = K'\left(\frac{du}{dy}\right)_{av}^{n'-1} \qquad \left(\frac{du}{dy}\right)_{av} = 11n \qquad N_{Re,n} = \frac{n^{2-n'}D_a^2\rho}{11^{n'-1}K'}$$

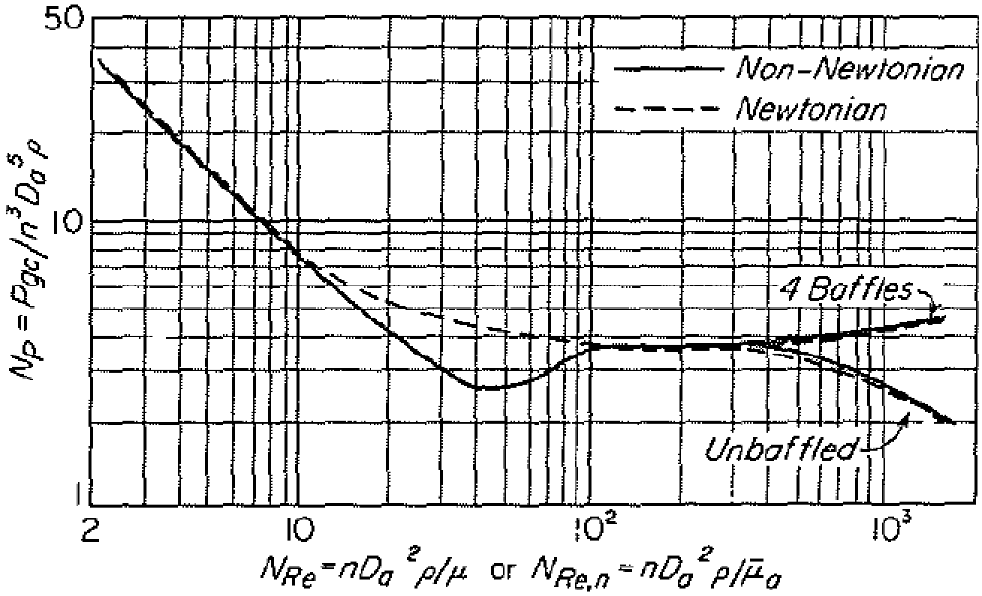


FIGURE 9.14
Power correlation for a six-blade turbine in non-newtonian liquids.

BLENDING AND MIXING

For a standard six-blade turbine:

$$q = 0.92nD_a^3\left(\frac{D_t}{D_a}\right) \tag{9.30}$$

$$t_T \approx \frac{5V}{q} = 5\frac{\pi D_t^2 H}{4}\frac{1}{0.92nD_a^2 D_t} \tag{9.31}$$

$$nt_T\left(\frac{D_a}{D_t}\right)^2\left(\frac{D_t}{H}\right) = \text{const} = 4.3 \tag{9.32}$$

A general correlation given by Norwood and Metzner is shown in Fig. 9.16. The Froude number in Eq. (9.33) implies some vortex effect, which may be present at low Reynolds numbers, but it is doubtful whether this term should be included for a baffled tank at high Reynolds numbers. When $N_{Re} > 10^5$, f_t is almost constant at a value of 5.

$$f_t = \frac{t_T(nD_a^2)^{2/3}g^{1/6}D_a^{1/2}}{H^{1/2}D_t^{3/2}} = nt_T\left(\frac{D_a}{D_t}\right)^2\left(\frac{D_t}{H}\right)^{1/2}\left(\frac{g}{n^2D_a}\right)^{1/6} \tag{9.33}$$

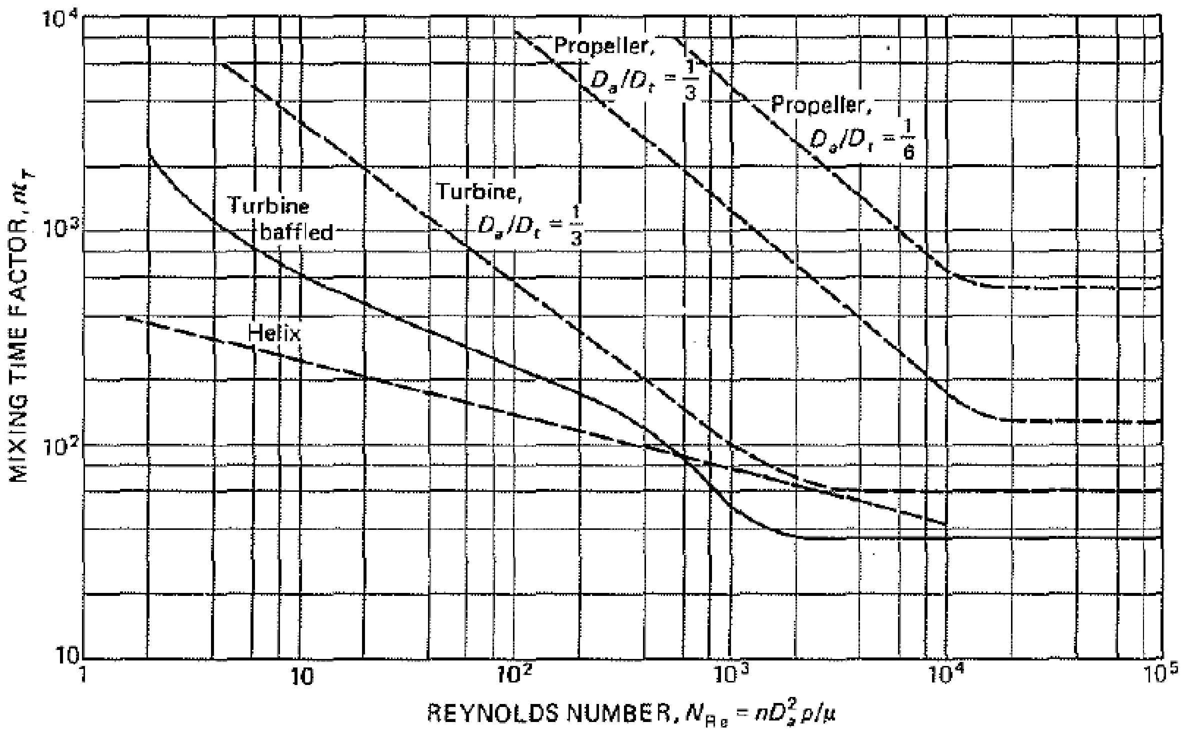


FIGURE 9.15
Mixing times in agitated vessels. Dashed lines are for unbaffled tanks; solid line is for an unbaffled tank.

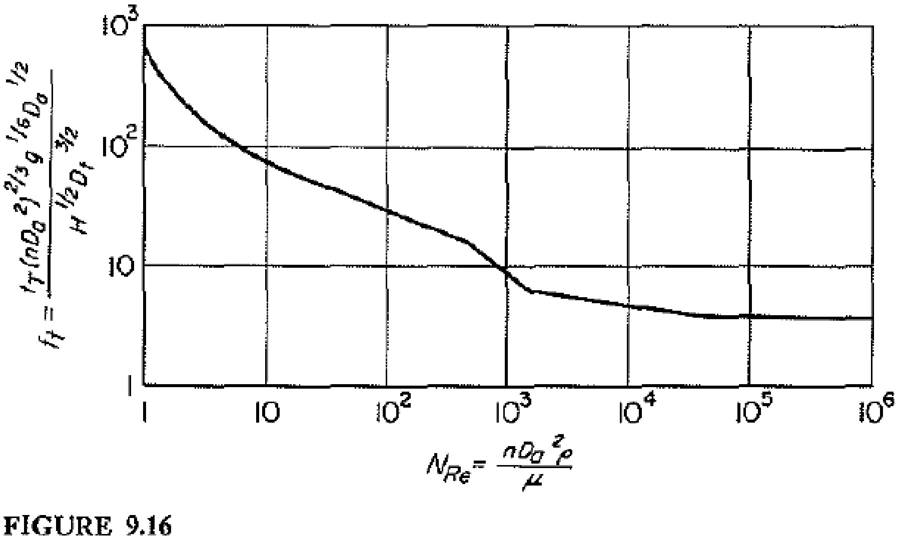


FIGURE 9.16
Correlation of blending times for miscible liquids in a turbine-agitated baffled vessel. (*After Norwood and Metzner.*[33])

The propeller data in Fig. 9.15 were taken from a general correlation of Fox and Gex,[16] whose mixing-time function differs from both Eqs. (9.32) and (9.33):

$$f_t' = \frac{t_T(nD_a^2)^{2/3}g^{1/6}}{H^{1/2}D_t} = nt_T\left(\frac{D_a}{D_t}\right)^{3/2}\left(\frac{D_t}{H}\right)^{1/2}\left(\frac{g}{n^2D_a}\right)^{1/6} \qquad (9.34)$$

Their data were for D_a/D_t of 0.07 to 0.18; the extrapolation to $D_a/D_t = \frac{1}{3}$ for Fig. 9.15 is somewhat uncertain.

JET MIXERS.

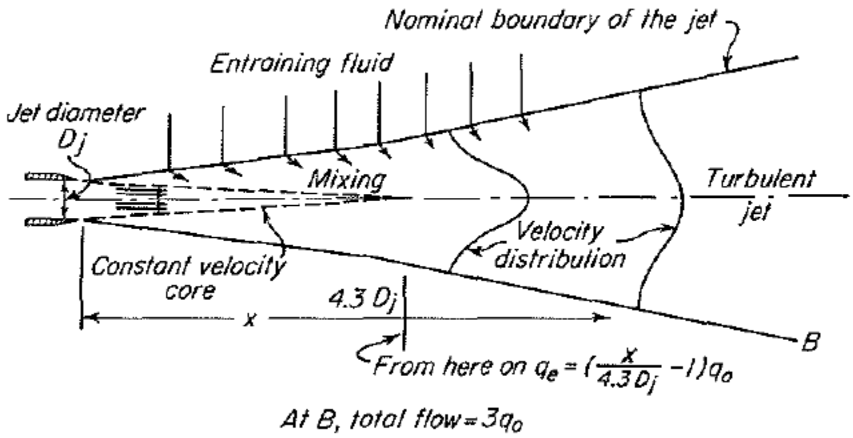


FIGURE 9.17
Flow of a submerged circular jet. (*After Rushton and Oldshue.*[43])

$$q_e = \left(\frac{X}{4.3D_j} - 1\right)q_0 \qquad (9.35)$$

where q_e = volume of liquid entrained per unit time at distance X from nozzle
q_0 = volume of liquid leaving jet nozzle per unit time

SUSPENSION OF SOLID PARTICLES

Nearly complete suspension with filleting.

Complete particle motion.

Complete suspension or complete off-bottom suspension.

Uniform suspension.

Zwietering's correlation is based on data for five types of impellers in six tanks from 6 in. to 2 ft in diameter. The critical stirrer speed is given by the dimensionless equation

$$n_c D_a^{0.85} = S \nu^{0.1} D_p^{0.2} \left(g \frac{\Delta\rho}{\rho} \right)^{0.45} B^{0.13} \qquad (9.36)$$

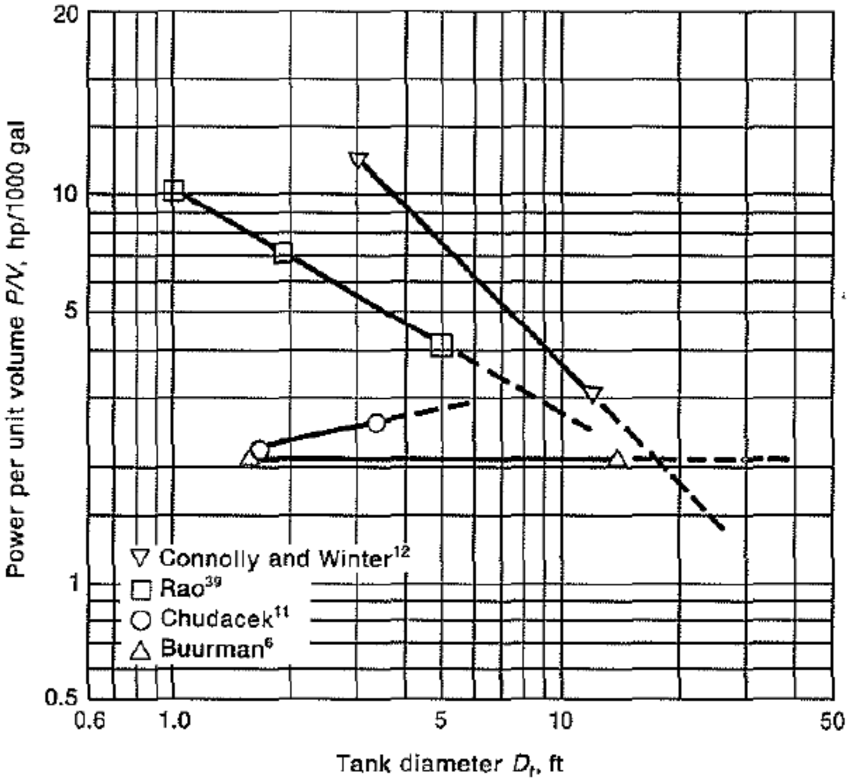
where n_c = critical stirrer speed
D_a = agitator diameter
S = shape factor
ν = kinematic viscosity
D_p = average particle size
g = gravitational acceleration
$\Delta\rho$ = density difference
ρ = liquid density
B = 100 × weight of solid/weight of liquid

Typical values of S are given in Table 9.4.

*(Most reliable for scale-up or for predicting the condition of suspebsion in the absence of experimental dara))

TABLE 9.4
Shape factor S in Eq. (9.36) for critical stirrer speed

Impeller type	D_t/D_a	D_t/E	S
Six-blade turbine	2	4	4.1
$D_a/W = 5$	3	4	7.5
$N_P = 6.2$	4	4	11.5
Two-blade paddle	2	4	4.8
$D_a/W = 4$	3	4	8
$N_P = 2.5$	4	4	12.5
Three-blade propeller	3	4	6.5
$N_P = 0.5$	4	4	8.5
	4	2.5	9.5



Rao: Six-blade turbine, $W/D_a = 0.3$

Chudacek: Six-blade turbine, $W/D_a = 0.2$

Buurman: Four-blade turbine, $W/D_a = 0.25$

$D_a/D_t = 1/3$

$E/D_t = 1/4$\

Sand-water

	Sand
D_p, μm	200
$\Delta\rho$, g/cm³	1.59
B	11.1

FIGURE 9.19
Power required for complete suspension of solids in agitated tanks using pitched-blade turbines.

DISPERSION OPERATIONS

CHARACTERISTICS OF DISPERSED PHASE; MEAN DIAMETER

$$\frac{\pi N D_p^3}{6} = \Psi \qquad \pi N D_p^2 = a \qquad D_p = \frac{6\Psi}{a}$$

volume-surface mean diameter or the *Sauter mean diameter*:

$$\overline{D}_s \equiv \frac{6\Psi}{a}$$

GAS DISPERSION; BUBBLE BEHAVIOR

$$F_b - F_g = \frac{g}{g_c}\frac{\pi D_p^3}{6}(\rho_L - \rho_V)$$

where ρ_L = density of liquid
ρ_V = density of vapor
F_b = total buoyant force
F_g = force of gravity

$$F_D = \pi D_o \sigma$$

$$D_p = \left[\frac{6 D_o \sigma g_c}{g(\rho_L - \rho_V)}\right]^{1/3}$$

where D_o = orifice diameter
σ = interfacial tension

GAS DISPERSION IN AGITATED VESSELS.

for gas dispersion in pure liquids by a six-blade turbine impeller.

For low gas holdups ($\Psi < 0.15$)

$D_s : 2 - 5mm$

$$\overline{D}_s' = 4.15 \frac{(\sigma g_c)^{0.6}}{(P g_c/V)^{0.4}\rho_L^{0.2}}\Psi^{1/2} + 0.9$$

Interfacial area (1/mm):

$$a' = 1.44 \frac{(Pg_c/V)^{0.4}\rho_L^{0.2}}{(\sigma g_c)^{0.6}}\left(\frac{\overline{V}_s}{u_t}\right)^{1/2}$$

Usually: $u_t = 0.2 m/s$

$$\Psi = \left(\frac{\overline{V}_s\Psi}{u_t}\right)^{1/2} + 0.216\frac{(Pg_c/V)^{0.4}\rho_L^{0.2}}{(\sigma g_c)^{0.6}}\left(\frac{\overline{V}_s}{u_t}\right)^{1/2}$$

In these three equations all quantities involving the dimension of length are in millimetres (mm).

where $\overline{V}_s$ = superficial velocity of gas
= volumetric gas feed rate divided by the cross-sectional area of vessel
u_t = bubble rise velocity in stagnant liquid

POWER INPUT TO TURBINE DISPERSERS.

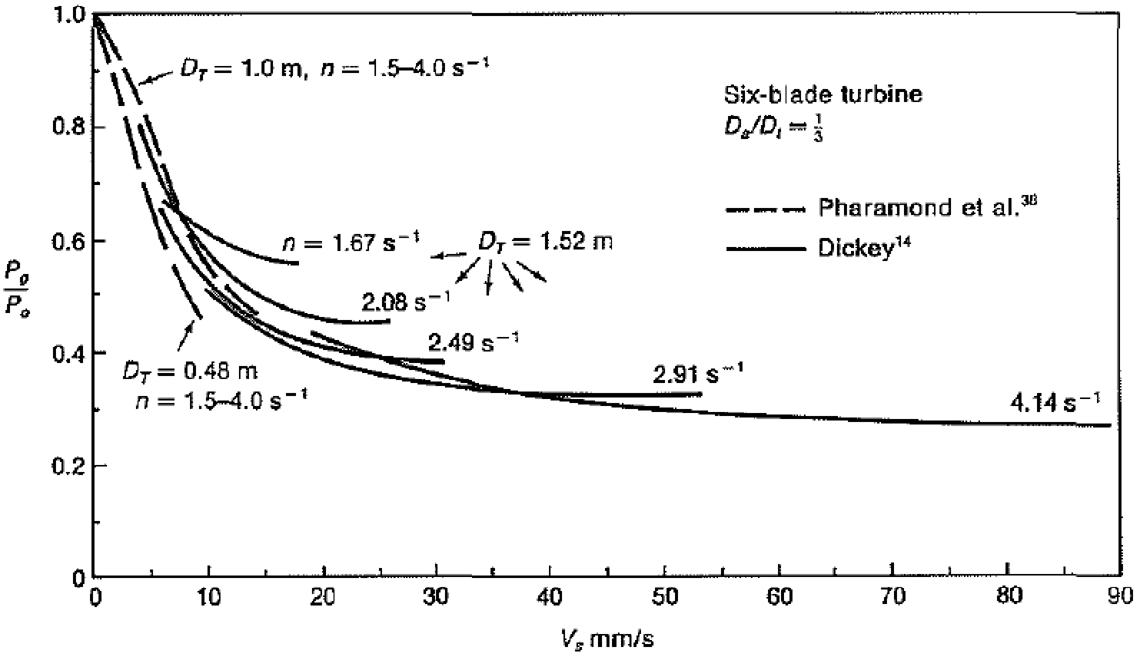


FIGURE 9.20
Power consumption in aerated turbine-agitated vessels.

GAS-HANDLING CAPACITY AND LOADING OF TURBINE IMPELLERS.
From data for tanks 1.54 and 0.29 m in diameter and velocities up to 75 mm/s,

$$\overline{V}_{s,c} = 0.114\left(\frac{P_g}{V}\right)\left(\frac{D_t}{1.5}\right)^{0.17}$$

(P_g/V) is in W/m³, D_t in m, and $\overline{V}_{s,c}$ in mm/s.

SCALEUP OF AGITATOR DESIGN.

In geometrically similar vessels:

$$\frac{P}{V} \propto n^3 D_a^{\,2}$$

$$\frac{(P/V)_1}{(P/V)_2} = \left(\frac{n_1}{n_2}\right)^3\left(\frac{D_{a1}}{D_{a2}}\right)^2$$

Flow Past Immersed Bodies

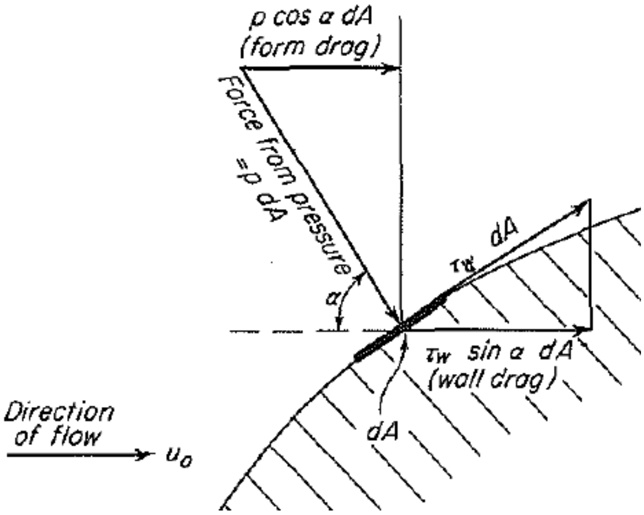


FIGURE 7.1
Wall drag and form drag on immersed body.

$$C_D \equiv \frac{F_D/A_p}{\rho u_0^2/2g_c}$$

A_p : The area obtained by projecting the body on a plane perpendicular to the direction of flow

EQUATIONS FOR ONE-DIMENSIONAL MOTION OF PARTICLE THROUGH FLUID.

$$F_e = \frac{ma_e}{g_c} \qquad F_b = \frac{m\rho a_e}{\rho_p g_c} \qquad F_D = \frac{C_D u_0^2 \rho A_p}{2g_c} \qquad a_e = g \quad or \quad a_e = r\omega^2$$

$$\frac{m}{g_c}\frac{du}{dt} = F_e - F_b - F_D \qquad \Rightarrow \qquad \frac{du}{dt} = a_e - \frac{\rho a_e}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m} = a_e \frac{\rho_p - \rho}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m}$$

Terminal velocity:

$$u_t = \sqrt{\frac{2g(\rho_p - \rho)m}{A_p \rho_p C_D \rho}} \qquad u_t = \omega\sqrt{\frac{2r(\rho_p - \rho)m}{A_p \rho_p C_D \rho}}$$

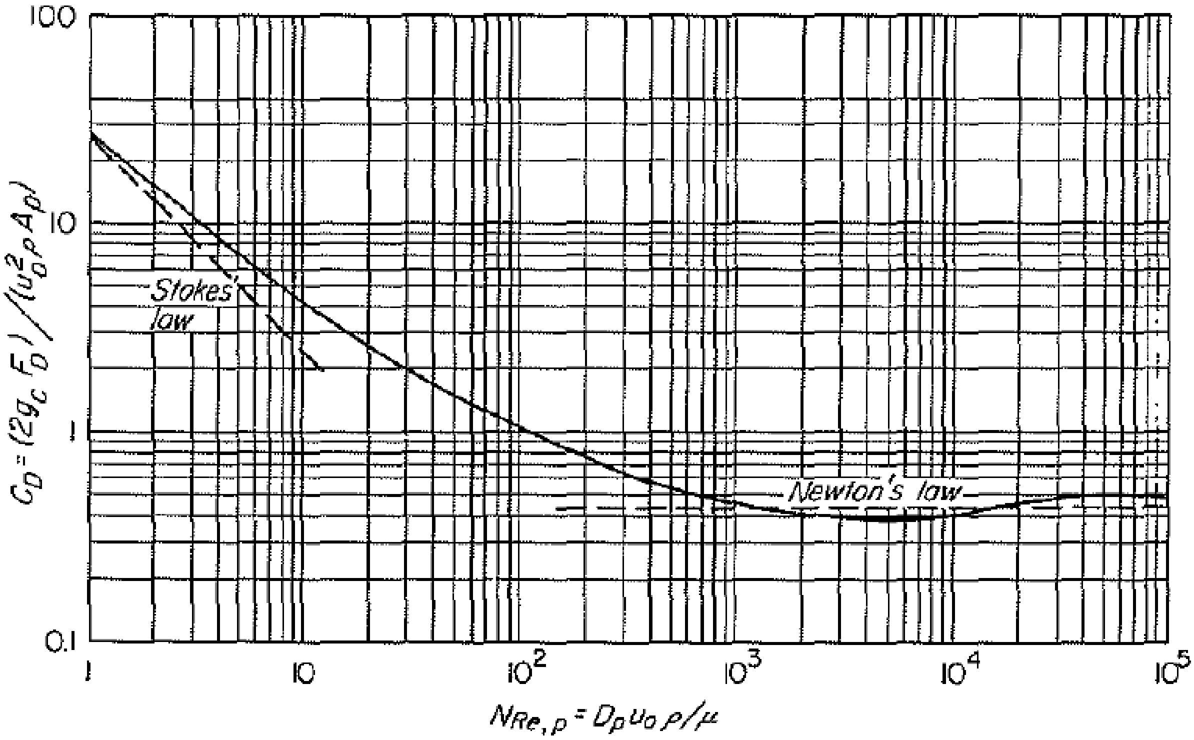


FIGURE 7.6
Drag coefficients for spheres.

MOTION OF SPHERICAL PARTICLES.

$$u_t = \sqrt{\frac{4g(\rho_p - \rho)D_p}{3C_D\rho}}$$

For $N_{\mathrm{Re},p} < 1.0$:

$$C_D = \frac{24}{N_{\mathrm{Re},p}} \qquad F_D = \frac{3\pi\mu u_t D_p}{g_c} \qquad u_t = \frac{gD_p^2(\rho_p - \rho)}{18\mu}$$ Stoke's law

For $1000 < N_{\mathrm{Re},p} < 200{,}000$:

$$C_D = 0.44 \qquad F_D = \frac{0.055\pi D_p^2 u_t^2 \rho}{g_c} \qquad u_t = 1.75\sqrt{\frac{gD_p(\rho_p - \rho)}{\rho}}$$ Newton's Law

CRITERION FOR SETTLING REGIME.

$$K = D_p\left[\frac{g\rho(\rho_p - \rho)}{\mu^2}\right]^{1/3}$$

For $K < 2.6$: Stoke's Law
For $68.9 < K < 2360$: Newton's Law

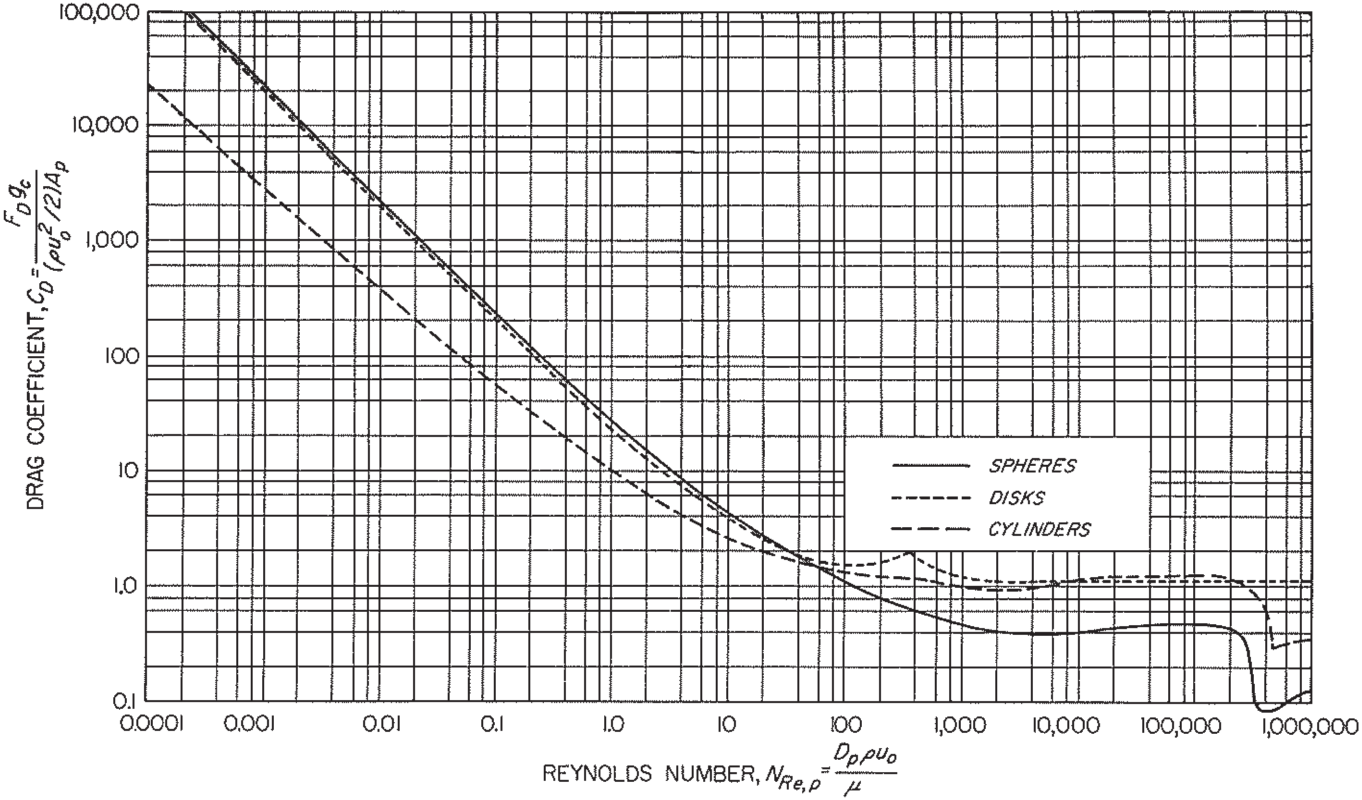


FIGURE 7.3
Drag coefficients for spheres, disks, and cylinders. [*By permission from J. H. Perry (ed.), Chemical Engineers' Handbook, 6th ed., p. 5–64. Copyright, © 1984, McGraw-Hill Book Company.*]

FRICTION IN FLOW THROUGH BEDS OF SOLIDS

$\Phi_s = (6/D_p)/(s_p/v_p)$

TABLE 28.1
Sphericity of miscellaneous materials†

Material	Sphericity	Material	Sphericity
Spheres, cubes, short cylinders ($L = D_p$)	1.0	Ottawa sand	0.95
		Rounded sand	0.83
Raschig rings ($L = D_p$)		Coal dust	0.73
$L = D_o, D_i = 0.5D_o$	0.58‡	Flint sand	0.65
$L = D_o, D_i = 0.75D_o$	0.33‡	Crushed glass	0.65
Berl saddles	0.3	Mica flakes	0.28

† By permission, from J. H. Perry (ed.), *Chemical Engineers' Handbook*, 6th ed., p. 5-54, McGraw-Hill Book Company, New York, 1984.
‡ Calculated value.

For granular solids, Φ_s ranges from 0.6 to 0.95.

$$D_{eq} = \tfrac{2}{3}\Phi_s D_p \frac{\varepsilon}{1-\varepsilon}$$

For the typical void fraction of 0.4, $D_{eq} = 0.44\Phi_s D_p$, or the equivalent diameter is roughly half the particle size.

$$\overline{V} = \frac{\overline{V}_0}{\varepsilon}$$ average velocity in the channels $\overline{V}$

superficial or empty-tower velocity $\overline{V}_0$

the *Kozeny-Carman* equation
$$\frac{\Delta p}{L} = \frac{150\overline{V}_0\mu}{g_c\Phi_s^2 D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}$$ $N_{\text{Re},p} < 1$

the *Ergun* equation
$$\frac{\Delta p}{L} = \frac{150\overline{V}_0\mu}{g_c\Phi_s^2 D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{1.75\rho\overline{V}_0^2}{g_c\Phi_s D_p}\frac{1-\varepsilon}{\varepsilon^3}$$ $10 < N_{\text{Re},p} < 1000$

the Burke-Plummer equation
$$\frac{\Delta p}{L} = \frac{1.75\rho\overline{V}_0^2}{g_c\Phi_s D_p}\frac{1-\varepsilon}{\varepsilon^3}$$ $N_{\text{Re},p} > 1000$

MIXTURES OF PARTICLES:

$$\overline{D}_s = \frac{\sum_{i=1}^{n} N_i D_{pi}^3}{\sum_{i=1}^{n} N_i D_{pi}^2} \qquad \overline{D}_s = \frac{1}{\sum_{i=1}^{n} \frac{x_i}{D_{pi}}}$$

TABLE 7.1
Void fractions for dumped packings

D_p/D_t	ε for spheres	ε for cylinders
0	0.34	0.34
0.1	0.38	0.35
0.2	0.42	0.39
0.3	0.46	0.45
0.4	0.50	0.53
0.5	0.55	0.60

HINDERED SETTLING.

$$u_s = u_t(\varepsilon)^n$$

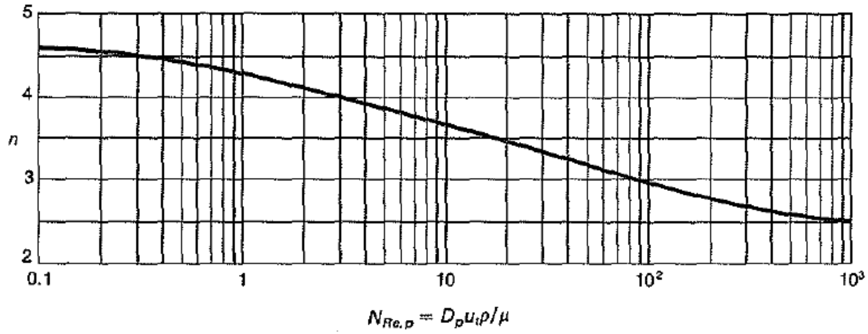


FIGURE 7.7
Plot of exponent n versus $N_{Re,p}$

For suspensions of free-flowing solid particles, the effective viscosity μ_s may be estimated from the relation[18]

$$\frac{\mu_s}{\mu} = \frac{1 + 0.5(1 - \varepsilon)}{\varepsilon^4} \tag{7.47}$$

Equation (7.47) applies only when $\varepsilon > 0.6$ and is most accurate when $\varepsilon > 0.9$.

SETTLING AND RISE OF BUBBLES AND DROPS.

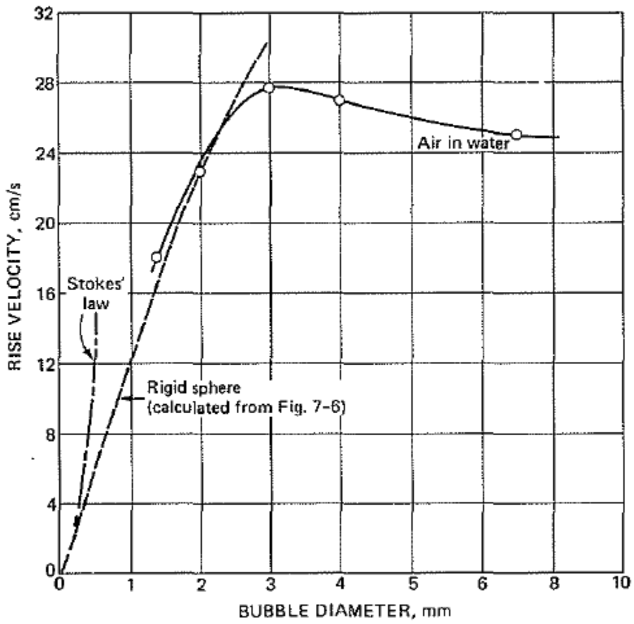


FIGURE 7.8
Rise velocity of air bubbles in water at 70°F. [*By permission, data taken from J. L. L. Baker and B. T. Chao, AIChE J., 11:268 (1965).*]

FLUIDIZATION

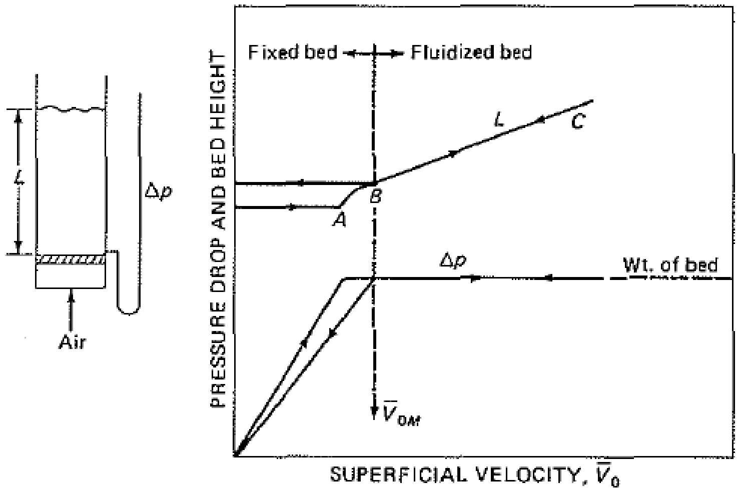


FIGURE 7.9
Pressure drop and bed height vs. superficial velocity for a bed of solids.

MINIMUM FLUIDIZATION VELOCITY.

Generally:

$$\Delta p = \frac{g}{g_c}(1-\varepsilon)(\rho_p-\rho)L$$

$$\frac{\Delta p\, g_c}{L} = \frac{150\mu\overline{V}_0}{\Phi_s^2 D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{1.75\rho\overline{V}_0^2(1-\varepsilon)}{\Phi_s D_p \varepsilon^3}$$

for the minimum fluidization velocity $\overline{V}_{0M}$:

$$\frac{\Delta p}{L} = \frac{g}{g_c}(1-\varepsilon_M)(\rho_p-\rho)$$

$$\frac{150\mu\overline{V}_{0M}}{\Phi_s^2 D_p^2}\frac{(1-\varepsilon_M)}{\varepsilon_M^3} + \frac{1.75\rho\overline{V}_{0M}^2}{\Phi_s D_p}\frac{1}{\varepsilon_M^3} = g(\rho_p-\rho)$$

For very small particles, only the laminar-flow term of the Ergun equation is significant.

In the limit of very large sizes, the laminar-flow term becomes negligible

For small sphere particles or $N_{\mathrm{Re},p} < 1$:

$$\frac{u_t}{\overline{V}_{0M}} = \frac{8.33(1-\varepsilon_M)}{\Phi_s^2\varepsilon_M^3}$$

For large sphere particles or $1000 < N_{\mathrm{Re},p}$:

$$\frac{u_t}{\overline{V}_{0M}} = \frac{2.32}{\varepsilon_M^{3/2}}$$

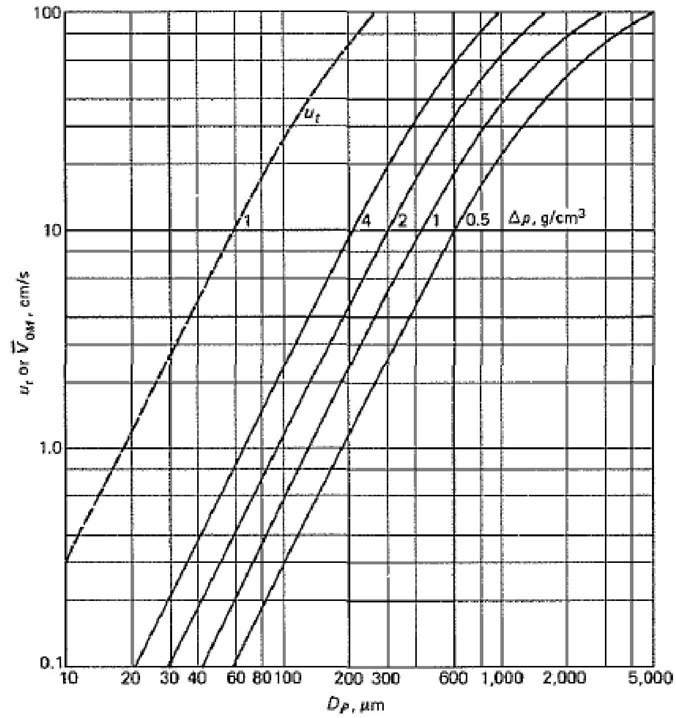


FIGURE 7.10
Minimum fluidization velocity and terminal velocity with air at 20°C and 1 atm ($\varepsilon_M = 0.50, \Phi_s = 0.8, \Delta\rho = \rho_p - \rho$).

TYPES OF FLUIDIZATION.:

Particulate fluidization.

Bubbling fluidization.

EXPANSION OF FLUIDIZED BEDS.:

$$\frac{\Delta p}{L} = \frac{g}{g_c}(1 - \varepsilon)(\rho_p - \rho)$$

Particulate fluidization.

Particulate fluidization. For particulate fluidization the expansion is uniform, and the Ergun equation, which applies to the fixed bed, might be expected to hold approximately for the slightly expanded bed.

Assuming the flow between the particles is laminar.:

$$\frac{\varepsilon^3}{1 - \varepsilon} = \frac{150\overline{V}_0\mu}{g(\rho_p - \rho)\Phi_s^2 D_p^2}$$

$$L = L_M \frac{1 - \varepsilon_M}{1 - \varepsilon}$$

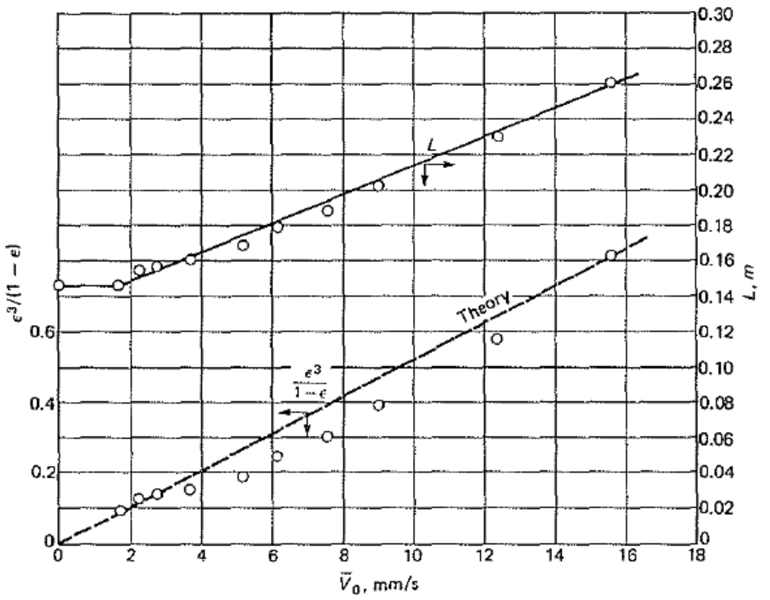


FIGURE 7.11
Bed expansion in particulate fluidization. [*By permission, data taken from R. H. Wilhelm and M. Kwauk, Chem. Eng. Prog., 44:201 (1948).*]

$$\overline{V}_0 = \varepsilon^m$$

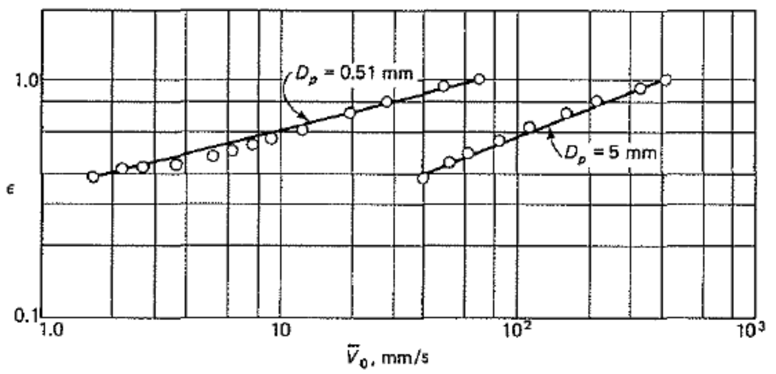


FIGURE 7.12
Variation of porosity with fluid velocity in a fluidized bed. [*By permission, data taken from R. H. Wilhelm and M. Kwauk, Chem. Eng. Prog., 44:201 (1948).*]

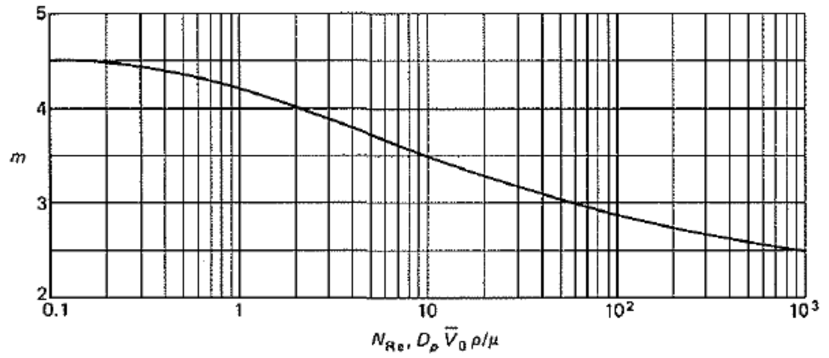


FIGURE 7.13
Exponent *m* in correlation for bed expansion [Eq. (7.59)]. (*By permission, from M. Leva, Fluidization, p. 89. Copyright, © 1959, McGraw-Hill Book Company.*)

Bubbling fluidization.

$\overline{V}_0 = f_b u_b + (1 - f_b)\overline{V}_{0M}$

where f_b = fraction of bed occupied by bubbles
u_b = average bubble velocity

$u_b \approx 0.7\sqrt{gD_b}$

$L_M = L(1 - f_b)$

$$\frac{L}{L_M} = \frac{u_b - \overline{V}_{0M}}{u_b - \overline{V}_0}$$

Compressible Flow

Perfect-gas Relationships.

$c_p = c_v + R$ $\qquad h = u + p/\rho$ $\qquad T\,ds = du + p\,d\frac{1}{\rho}$ $\qquad k = \frac{c_p}{c_v}$

$$s_2 - s_1 = c_v \ln\left[\frac{T_2}{T_1}\left(\frac{\rho_1}{\rho_2}\right)^{k-1}\right]$$

For isentropic processes:

$\frac{p}{\rho^k} = \text{constant}$ & $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} = \left(\frac{\rho_2}{\rho_1}\right)^{k-1}$

The *polytropic* process

$\frac{p}{\rho^n} = \text{constant}$

The bulk modulus of elasticity:

V is the volume of fluid subjected to the pressure change dp

$$K = -\frac{dp}{dV/V}$$

Mach number:

$\mathbf{M} = \frac{V}{c}$ $\qquad c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{K}{\rho}}$

Isentropic Flow.

Euler's equation $\qquad V\,dV + \frac{dp}{\rho} = 0$

the continuity equation: $\qquad \rho A V = \text{constant}$

$$\frac{dA}{dV} = \frac{A}{V}\left(\frac{V^2}{c^2} - 1\right) = \frac{A}{V}(\mathbf{M}^2 - 1)$$

The assumptions underlying this equation are that the flow is steady and frictionless.

$\frac{V^2}{2} + \frac{k}{k-1}\frac{p_1}{\rho_1{}^k}\rho^{k-1} = \text{constant}$ $\qquad \frac{V_1{}^2}{2} + \frac{k}{k-1}\frac{p_1}{\rho_1} = \frac{V_2{}^2}{2} + \frac{k}{k-1}\frac{p_2}{\rho_2}$

For adiabatic flow from a reservoir where conditions are given by p_0, ρ_0, T_0, at any other section

$\frac{V^2}{2} = \frac{kR}{k-1}(T_0 - T)$ $\qquad \mathbf{M}^2 = \frac{V^2}{c^2} = \frac{2kR(T_0 - T)}{(k-1)kRT} = \frac{2}{k-1}\left(\frac{T_0}{T} - 1\right)$

$\frac{T_0}{T} = 1 + \frac{k-1}{2}\mathbf{M}^2$ $\qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\mathbf{M}^2\right)^{k/(k-1)}$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2}\mathbf{M}^2\right)^{1/(k-1)}$$

Flow conditions are termed critical at the throat section when the velocity there is sonic. Sonic conditions are marked with an asterisk.

$\mathbf{M} = 1$: $c^* = V^* = \sqrt{kRT^*}$.

$\frac{V^*}{V} = \frac{1}{\mathbf{M}}\sqrt{\frac{T^*}{T}} = \frac{1}{\mathbf{M}}\sqrt{\frac{T^*}{T_0}}\sqrt{\frac{T_0}{T}} = \frac{1}{\mathbf{M}}\left\{\frac{1 + [(k-1)/2]\mathbf{M}^2}{(k+1)/2}\right\}^{\frac{1}{2}}$ $\qquad \frac{\rho^*}{\rho} = \frac{\rho^*}{\rho_0}\frac{\rho_0}{\rho} = \left\{\frac{1 + [(k-1)/2]\mathbf{M}^2}{(k+1)/2}\right\}^{1/(k-1)}$

$$\frac{A}{A^*} = \frac{1}{\mathbf{M}}\left\{\frac{1 + [(k-1)/2]\mathbf{M}^2}{(k+1)/2}\right\}^{(k+1)/2(k-1)}$$

$$\dot{m}_{max} = \rho^* A^* V^* = \rho_0\left(\frac{2}{k+1}\right)^{1/(k-1)} A^* \sqrt{\frac{kR2T_0}{k+1}}$$

For subsonic flow throughout a converging-diverging duct, the velocity at the throat must be less than sonic velocity, or $\mathbf{M}_t < 1$ with subscript t indicating the throat section. The mass rate of flow $\dot{m}$ is obtained from

$$\dot{m} = \rho V A = A \sqrt{2 p_0 \rho_0 \frac{k}{k-1} \left(\frac{p}{p_0}\right)^{2/k} \left[1 - \left(\frac{p}{p_0}\right)^{(k-1)/k}\right]} \qquad (6.3.24)$$

$$\frac{p_t}{p_0} \geq \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

For maximum mass flow rate, the flow downstream from the throat may be either supersonic or subsonic, depending upon the downstream pressure.

$$\left(\frac{p}{p_0}\right)^{2/k} \left[1 - \left(\frac{p}{p_0}\right)^{(k-1)/k}\right] = \frac{k-1}{2} \left(\frac{2}{k+1}\right)^{(k+1)/(k-1)} \left(\frac{A^*}{A}\right)^2$$

Shock Waves.

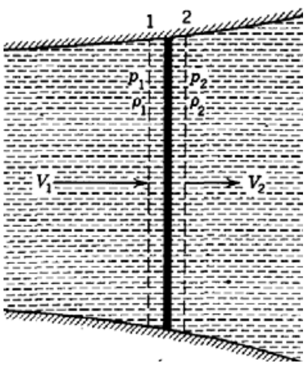


FIG. 6.2. Normal compression shock wave.

Continuity: $$G = \frac{\dot{m}}{A} = \rho_1 V_1 = \rho_2 V_2$$

Energy: $$\frac{V_1^2}{2} + h_1 = \frac{V_2^2}{2} + h_2 = h_0 = \frac{V^2}{2} + \frac{k}{k-1}\frac{p}{\rho}$$

momentum $$(p_1 - p_2)A = \rho_2 A V_2^2 - \rho_1 A V_1^2$$

$$M_1 . M_2 = 1$$

the Rankine-Hugoniot equations :

$$\frac{p_2}{p_1} = \frac{[(k+1)/(k-1)](\rho_2/\rho_1) - 1}{[(k+1)/(k-1)] - \rho_2/\rho_1}$$

$$\frac{\rho_2}{\rho_1} = \frac{1 + [(k+1)/(k-1)]p_2/p_1}{[(k+1)/(k-1)] + p_2/p_1} = \frac{V_1}{V_2}$$

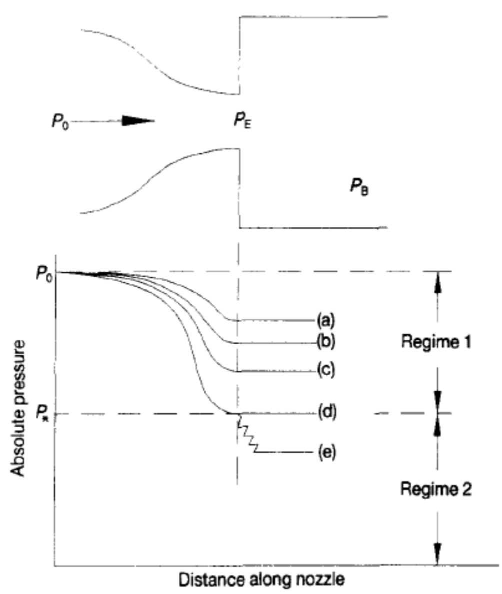
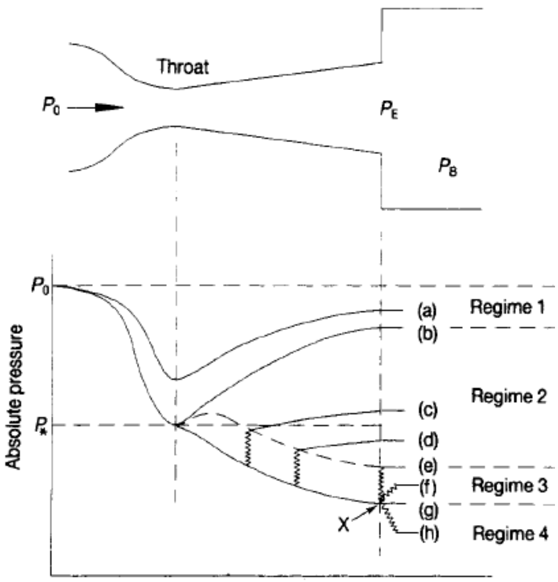
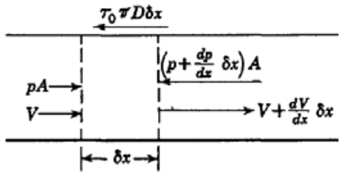


Figure
Pressure profiles for compressible flow through a convergent nozzle



Pressure profiles for compressible flow through a convergent–divergent nozzle

Adiabatic Flow with Friction in Conduits.



$$\frac{dp}{p} + \frac{f}{2D}\frac{\rho V^2}{p}\,dx + \frac{\rho V}{p}\,dV = 0$$

$$V^2 = \mathbf{M}^2 \frac{kp}{\rho}$$

$$\frac{dT}{T} = -\mathbf{M}^2(k-1)\frac{dV}{V} \qquad \frac{dV}{V} = \frac{d\mathbf{M}/\mathbf{M}}{[(k-1)/2]\mathbf{M}^2+1} \qquad \frac{dp}{p} = -\frac{(k-1)\mathbf{M}^2+1}{[(k-1)/2]\mathbf{M}^2+1}\frac{d\mathbf{M}}{\mathbf{M}}$$

$$\frac{f}{D}\,dx = \frac{2(1-\mathbf{M}^2)}{k\mathbf{M}^3\{[(k-1)/2]\mathbf{M}^2-1\}}\,d\mathbf{M} = \frac{2}{k}\frac{d\mathbf{M}}{\mathbf{M}^3} - \frac{k+1}{k}\frac{d\mathbf{M}}{\mathbf{M}\{[(k-1)/2]\mathbf{M}^2+1\}}$$

To avoid shock wave: M=1

Frictionless Flow through Ducts with Heat Transfer.

Continuity: $G = \frac{\dot{m}}{A} = \rho V$

Momentum $p + \rho V^2 = \text{constant}$

Energy:
$$q_H = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$
$$= c_p(T_{02} - T_{01})$$

T_{01} and T_{02} are the isentropic stagnation temperatures, i.e., the temperature produced at a section by bringing the flow isentropically to rest.

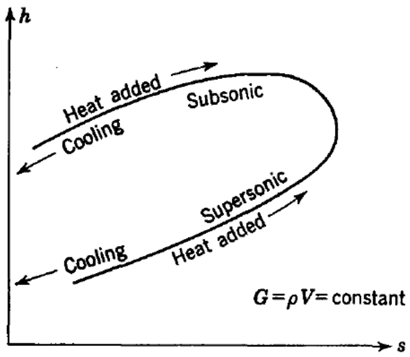


FIG. 6.5. Rayleigh line.

$$\frac{p_1}{p_2} = \frac{1 + k\mathbf{M}_2^2}{1 + k\mathbf{M}_1^2}$$

$$\frac{T_{01}}{T_1} = 1 + (k-1)\frac{\mathbf{M}_1^2}{2}$$

$$\frac{T_{02}}{T_2} = 1 + (k-1)\frac{\mathbf{M}_2^2}{2}$$

$$\frac{T_1}{T_2} = \left(\frac{\mathbf{M}_1}{\mathbf{M}_2}\frac{1 + k\mathbf{M}_2^2}{1 + k\mathbf{M}_1^2}\right)^2$$

$$\frac{T_{01}}{T_{02}} = \left(\frac{\mathbf{M}_1}{\mathbf{M}_2}\frac{1 + k\mathbf{M}_2^2}{1 + k\mathbf{M}_1^2}\right)^2 \frac{2 + (k-1)\mathbf{M}_1^2}{2 + (k-1)\mathbf{M}_2^2}$$

Steady, Isothermal Flow in Long Pipelines.

Equation of state:	$\frac{p}{\rho} = \text{constant} \qquad \frac{dp}{p} = \frac{d\rho}{\rho}$
Momentum	$\frac{dp}{p} + \frac{f}{2D}\frac{\rho V^2}{p}dx + \frac{\rho V}{p}dV = 0$
Continuity:	$\rho V = \text{constant}$
Energy	$T_0 = T\left[1 + \frac{(k-1)}{2}\mathbf{M}^2\right]$
Stagnation pressure	$p_0 = p\left(1 + \frac{k-1}{2}\mathbf{M}^2\right)^{k/(k-1)}$

in which p_0 is the pressure (at the section of p and $\mathbf{M}$) obtained by reducing the velocity to zero isentropically.

$$V = c\mathbf{M} = \sqrt{kRT}\,\mathbf{M} \qquad \frac{dV}{V} = \frac{d\mathbf{M}}{\mathbf{M}} = \frac{d\mathbf{M}^2}{2\mathbf{M}^2}$$

$$\frac{\rho V}{p}dV = \frac{V\,dV}{RT} = \frac{c^2}{RT}\mathbf{M}\,d\mathbf{M} = k\mathbf{M}\,d\mathbf{M}$$

$$\frac{\rho V^2}{p} = \frac{c^2\mathbf{M}^2}{RT} = k\mathbf{M}^2$$

$$\frac{dp}{p} = \frac{d\rho}{\rho} = -\frac{dV}{V} = -\frac{1}{2}\frac{d\mathbf{M}^2}{\mathbf{M}^2} = -\frac{k\mathbf{M}^2}{1-k\mathbf{M}^2}\frac{f\,dx}{2D}$$

$$\frac{dT_0}{T_0} = \frac{k-1}{2+(k-1)\mathbf{M}^2}d\mathbf{M}^2 \qquad \frac{dT_0}{T_0} = \frac{k(k-1)\mathbf{M}^4}{(1-k\mathbf{M}^2)[2+(k-1)\mathbf{M}^2]}\frac{f\,dx}{D}$$

$$\frac{dp_0}{p_0} = \frac{2-(k+1)\mathbf{M}^2}{2+(k-1)\mathbf{M}^2}\frac{k\mathbf{M}^2}{k\mathbf{M}^2-1}\frac{f\,dx}{2D} \qquad \frac{f}{D}L_{max} = \frac{1-k\mathbf{M}^2}{k\mathbf{M}^2} + \ln(k\mathbf{M}^2)$$

The superscript $*t$ indicates conditions at $\mathbf{M} = 1/\sqrt{k}$, and $\mathbf{M}$ and p represent values at any upstream section.

$$\frac{p^{*t}}{p} = \sqrt{k}\,\mathbf{M} \qquad \frac{V^{*t}}{V} = \frac{1}{\sqrt{k}\,\mathbf{M}}$$

Fluid measurement

Bernoulli's equation	$\frac{v_t^2}{2g} + \frac{p}{\gamma} + z = const.$